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W.H. Fleming's work during this period concerned risk sensitive stochastic control, and related questions about differential games. This theory provides a link between stochastic and deterministic (robust control) approaches to disturbance attenuation problems. H.J.Kushner's work developed efficient, general stochastic approximation methods for improving the operation of continuous or discrete event dynamical systems over a long time period. Applications to communication problems include large controlled multiplexing systems, which are approximated by diffusion type processes. The method yields a very efficient way of approximation as well as good numerical methods.

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Final Technical Report

This is a summary of research by W. H. Fleming and H. J.Kushner done under AFOSR Grant F49620-92-J-0081 "Stochastic Control and Nonlinear Estimation".

W. H. Fleming worked on risk sensitive control together with related questions in robust nonlinear control and differential games. Risk sensitive control theory provides a link between deterministic and stochastic approaches to disturbance attenuation problems. It uses ideas from the theory of large deviations for stochastic processes.

In work with W. M. McEneaney, risk sensitive control problems were considered on both finite and infinite time horizons for nonlinear systems described by stochastic differential equations. Logarithmic transformations were applied to the associated optimal cost functions for finite-time horizon problems. The value function for a zero-sum, two-controller differential game was obtained in the limit, as a small parameter (which represents noise intensity) tends to zero. Convergence to the value function of the differential game was proved by viscosity solution methods for nonlinear partial differential equations.

For risk sensitive control on an infinite time horizon, logarithmic transformations lead to stochastic differential games with an ergodic (average cost per unit time) payoff criterion. The value of the stochastic differential game is an optimal long-term growth rate of expected exponential cost, or equivalently an optimal Donsker-Varadhan large deviations rate.

To obtain results about robust nonlinear control, the noise intensity for the infinite horizon risk sensitive problem was made to tend to zero. A crucial question turned out to be whether, in this deterministic limit, the optimal long-term growth rate is zero or positive. If it is zero, then a dissipation inequality which plays a key role in robust nonlinear control theory holds.

In work with M. J. James the dependence of the risk-sensitive index (i.e. the optimal long term growth rate) on an additional small parameter was examined. This parameter corresponds to the reciprocal of an operator norm bound familiar in robust /H-infinity control. Depending on the relative sizes of this parameter and another parameter indicating noise, a mixture of H_2 and H_∞ norms for nonlinear systems is obtained.

The ideas outlined above are being explored via numerical experiments. A risk sensitive control formulation of a model for semi-active vehicle suspension was used to illustrate the method.

Fleming and Soner completed a research monograph on "Controlled Markov Processes and Viscosity Solutions".

H. J. Kushner's work covered many aspects of modern stochastic control. Methods for stochastic approximation with averaging of the iterates which yield optimal rates of convergence were developed. These results greatly reduce the traditional difficulty of selecting good step size sequences. They yield asymptotic results which are equivalent to what we would get if we used the optimal matrix valued gain for the step sizes.

A major accomplishment was the completion of the book on new and very powerful numerical methods in stochastic control. It covers the great bulk of the formulations of the continuous time problems which have appeared to date, as well as newer and less well known formulations: reflecting boundary problems (for example from the heavy traffic approximations), singular controls, ergodic problems, etc. These methods are becoming the numerical methods of choice for stochastic control problems in continuous time.

We developed an effective method for the modelling and optimal control of large trunk line systems under heavy traffic as well as for the numerical solution. The controls are decisions concerning rerouting, and pose serious non standard difficulties. The optimal costs for the network are well approximated by optimal costs for the heavy traffic limit.

Numerical solutions show that the methods actually do provide excellent strategies in realistic situations. These results show the great power of modern methods in stochastic control for the treatment of difficult and very practical problems.

Large controlled multiplexing systems are approximated by diffusion type processes. This yields a very efficient way of approximation as well as good numerical methods. The "limit" equations are an efficient aggregation of the original system, and they provide the basis of the good numerical approximations for the control problem. The numerical approximations have the structure of the original problem, but are generally much simpler. The control can occur in a variety of places; e.g., the type of "leaky bucket" controllers, control of "marked cells" at the transmitter buffer, etc. These are equivalent. Various forms of the optimal control problem have been explored, where the aim was to control or balance the losses at the control with those due to buffer overflow. These are typical of many possibilities. The extensive numerical experiments show that much can be saved via the use of optimal controls or reasonable approximations to them. We discuss systems with several classes of sources, various aggregation methods and control approximation schemes. The results show that the approach is a very useful tool for providing both qualitative and quantitative information on problems in ATM and broadband integrated data networks. This would be hard to get otherwise, and amply demonstrates the power of modern techniques in stochastic control for the effective treatment of problems of great interest and complexity. We developed efficient and general stochastic approximation (SA) methods for improving the operation of parametrized systems of either the continuous or discrete event dynamical systems types. The emphasis was on systems which operate over a very long time period. The number of applications is increasing at a rapid rate. This is partly due to the increasing activity in computing pathwise derivatives and adapting them to the average cost problem. The powerful ODE type methods have been extended in a fairly general context, based on weak convergence ideas. The results and proof techniques are applicable to a wide variety of applications. Exploiting the full potential of these ideas can greatly simplify and extend much current work. The breadth and relative ease of using the basic ideas is illustrated through typical examples from discrete event dynamical systems, piecewise deterministic dynamical systems, and stochastic differential equations models. The algorithms for distributed/asynchronous updating as well as the fully synchronous schemes were developed.

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